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Algebraic Thinking Ability of class 7 SMP on Material Algebraic Form

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Abstract. Algebraic thinking ability is one of the mathematical thinking abilities that students must have because algebraic skills are needed in other materials, such as exponents, logarithms, trigonometry and so on. The purpose of this study was to determine the algebraic thinking skills of 7th grade middle school students in solving algebra problems. The research method used is a qualitative descriptive approach. The research instrument used was an algebraic thinking ability and interview test. The results of the analysis of this study indicate that generational, transformational, and global meta-level skills have been seen in some 7th grade students. Students who are in high-level groups have high generational, transformational, and meta-global abilities, students in moderate groups have high generational capability, moderate transformational, and low to moderate global meta-level. Whereas the ability of students in low groups has low to moderate generational abilities, moderate transformational and low global meta-levels.

Keywords : Algebraic thinking ability, Algebraic form

INTRODUCTION ~ Mathematics is one of the important subjects, so it is taught from elementary to tertiary level. Students are expected to be able to solve social, economic and several other problems after studying mathematics. Mathematics according to Wittgenstein (1991) is a way to find answers to problems faced by humans; a way of using information, using knowledge and measurement shapes, using knowledge about counting. In this case it seems that mathematics is important for students to learn, because the benefits of mathematics affect the development that occurs in the daily lives of students themselves.

Mathematics according to Uno (2007) is a field of science in the form of tools for thinking, communicating, tools for solving problems in various practical problems, and having branches including arithmetic, algebra, geometry and analysis. Each branch of mathematics is studied in schools and is at a stage that was previously set by the curriculum. One branch of mathematics that will be discussed in this study is algebra.

Algebra according to NCTM (National Council of Mathematic Teachers) is a branch of mathematics that uses mathematical statements to describe relationships of various things. In this case algebra is a tool to generalize and solve mathematical problems. In the opinion of I. N. Hersterin (1964) algebra is the study of mathematical symbols and rules for manipulating these rules. Based on the 2013 curriculum in Indonesia, algebra material was introduced at the beginning of the seventh grade of the odd semester. In its basic competence students are required to be able to explain the form of algebra and its elements, perform

algebraic form operations, and be able to solve problems related to algebraic forms

Algebra studies at very young age help students to understand mathematical structure and generality (Blanton & Kaput, 2011). The idea of NCTM (1998) establishes a solution for the transition from arithmetic to algebra by introducing algebra to the lower classes through finding patterns and generalizing.

According to NCTM (National Council of Teachers of Mathematics) (2000) one of the algebraic abilities that must be possessed by students in secondary schools is being able to present and analyze various patterns in the form of words, tables, graphics, and with symbolic rules.

Algebra is a way to express generalizations about numbers, quantities, relations and functions (Watson, 2007). Algebra is a new process in solving mathematical problems that students must learn and is the basis for further material. This is reinforced by the statement of Kriegler (2001) who said that algebra is a gateway to further understanding mathematics.

Traditionally, algebra has been started at secondary level because of at least three reasons: algebra emerged after historical arithmetic, psychological readiness on learning abstract like algebra, and difficulties that students have with algebra (Carraher, Schliemann, Brizuela, & Earnest, 2006). Some junior high school students still experience difficulties in understanding algebra material and utilizing algebra in solving problems. This shows the results that are contrary to expectations on the basic competencies that have been set.

Strengthened by the study of Qur'ani (2015) with the results of the study stating that there are some students at the junior secondary level who lack mastering algebra material. This is especially true for 7th grade students who are in the process of transitioning from arithmetic to algebraic thinking. Hidayanto, et al (2014) stated that the transition process is divided into several forms: (1) finding a calculation pattern, namely in the transition process of thinking students find a certain pattern of calculations done, (2) finding a relationship (relation) in a pattern, that is finding a certain pattern from a calculation, and (3) symbolizing, writing a symbol in accordance with his wishes.

The view that developed in society about algebra namely algebra is a domain that makes mathematics into an incomprehensible world, because in this domain began to be introduced variables and various symbols in completing mathematical operations (Warren, 2003)

Factors that cause students' difficulties in learning algebra include the concept of variables and symbols that they have never gotten in arithmetic learning in basic education so students have difficulty understanding the variable notation identified from algebraic learning outcomes (Jupri, Drijvers, Van den Heuvel-Panhuizen, 2014)

Lack of exposure to the ability of algebra which has an important role in the material in the form of trigonometry, geometry, statistics, and other material contained in mathematics and other subjects in school or in the outside world. The ability to operate algebraic forms cannot be separated from understanding, for example when learning equations, inequalities, functions, systems of equations and others. Algebra operations are mastered will help students in solving and understanding the next mathematical problem. This is also reinforced by Soedjadi's statement (1995: 27) which states that good algebraic abilities help someone in understanding mathematics.

Thinking is an activity that is often done by everyone. Thinking is a mental activity that involves the work of the brain. According to Slavin (1994) thinking occurs in every human mental activity that functions to formulate or solve problems, make decisions, and seek understanding. There are three basic ideas about thinking, namely (1) thinking is cognitive activity, which arises internally in the mind but can be predicted from behavior, (2) thinking is a process that involves some manipulation of knowledge in the cognitive system. New information is combined with stored knowledge so that it changes one's knowledge to something faced, (3) thinking is directed and produces behavior to solve problems (Mayer in Solso, 2007). So based on some of the opinions above it can be concluded that thinking is a mental activity that involves new information and knowledge possessed, aims to solve problems, make decisions and seek understanding.

Understanding algebra needs a different way of thinking because arithmetic thinking is not operated at the same level of abstraction as algebra, even though both involve written symbols and involve an understanding of operations. But the arithmetic symbol in the form of numbers and operations can directly be imagined how much is the quantity. While algebra uses symbols in the form of letters which are general forms of certain numbers and cannot be imagined how much is the quantity. This way of thinking is called algebraic thinking. Algebraic thinking according to Windsor (2008) is an important and fundamental element of mathematical thinking and mathematical reasoning.

Someone who has the ability to think algebra in solving a problem will be more advanced than someone who solves problems by thinking arithmetic. This is true because according to Van de Walle (2008: 4) students who rely on the relationship between numbers on the operation of signs (=) and not on direct calculations have gone one step further in their algebraic mindset.

Algebraic thinking according to Herbert and Brown (1997: 123) is a thought process that uses mathematical symbols and tools to analyze different solutions by 1) digging information from problems, 2) describing or showing information mathematically in the

form of words, diagrams, tables, graphs, and equations, 3) interpret and apply mathematical findings, such as solving unknowns, testing guesses, and looking for functional relationships of the same and new problems that are still related.

Algebraic thinking according to Kieran (2004) is a thought process that involves the development of ways of thinking using algebraic symbols as tools, but not separate from algebra, and also ways of thinking without using algebraic symbols. In working on algebraic questions students carry out activities consisting of:

1) Generational, including the formation of expressions and equations, including the formation of equations that contain an unknown quantity that represents the problem situation. Most of the activities of building the meaning of algebraic objects occur in generational algebraic activities.
2) Transformational, including gathering of similar terms, extending, factoring, adding, multiplying and replacing polynomial expressions, solving equations and simplifying expressions. Transformational has a lot to do with changing the form of expressions or equations to maintain equality.
3) Global meta-level, including problem solving, modeling, studying change, predicting, paying attention to structure, proving and analyzing relationships.

Three types of algebraic abilities proposed by Kieran (2004) were adopted as the basis for determining the classification of students' algebraic thinking abilities in this study, with indicators of each ability as follows:

Table 1. Indicators of Algebraic Thinking Ability

Type of Ability	Indicator
Generational	1. Determine the meaning of variables of a problem 2. Represents problems in the relationship between variables
Transformational	1. Perform algebraic form operations 2. Determine the completion of an equation in algebra
Global Meta-Level	1. Use algebra to analyze changes, relationships and predict problems in mathematics 2. Model the problem and solve it

METHOD

This research uses descriptive research method. According to Nawawi (2015: 67), descriptive method is a problem-solving procedure that is investigated by describing, describing the state of the object of the subject or object of one's research, institutions, society now based on facts that occur as they are.

Data collected in the form of written test results, interviews and documentation. This study aims to describe the algebraic thinking skills of seventh grade junior high school students on algebraic subjects. The

subjects of this study were grade VII students with 3 subjects, one person with high mathematical ability (T), one person with moderate ability (S), and one person with low ability (R). Subjects were taken based on the results of junior high school entrance tests, grade 6 report cards and suggestions from subject teachers. The research instrument was the researcher as the main instrument then the test questions and interview guidelines as supporting instruments. The test questions instrument consisted of 6 algebraic form operation questions.

The data obtained were obtained from written test results taken directly from students using test instruments in accordance with algebraic thinking indicators. Then the students were interviewed about the results of the data obtained from the written test results. Then the data obtained were analyzed and conclusions drawn about students' algebraic thinking based on the algebraic thinking indicators discussed earlier. Grouping research subjects is based on the average test scores of each indicator of algebraic thinking ability with the criteria as in table 2 of each ability as follows:

Table 2. Criteria for Grouping Research Subjects

Group	Value Interval
High level	$66{,}67 \leq \bar{x} \leq 100$
Medium level	$33{,}33 \leq \bar{x} \leq 66{,}67$
Low level	$0 \leq \bar{x} \leq 33{,}33$

RESULTS

Based on the indicators in this study, the algebraic thinking ability test refers to generational abilities, transformational abilities, and global meta-level capabilities. The algebraic thinking ability test provided consisted of 6 questions covering each of two questions for generational abilities, two questions for transformational abilities and two questions for global meta-level abilities. Questions number 1 and 2 contain generational capability indicators, questions number 3 and 4 contain transformational indicators, and questions number 5 and 6 contain global meta-level capability indicators. Students are given time to work on the questions for 60 minutes and observed by researchers. Students are not allowed to open books and ask friends when working on problems

In addition to the written test, interviews were also conducted on the results of tests that students had done. So that no data is missed, the researcher uses a voice recorder. Interviews were carried out during recess while written tests were carried out during the math class in the library. Written interview and test activities will be held on September 6, 2019.

Description of the results of students' algebra abilities based on their mathematical ability are as follows:

Description of the algebraic thinking ability test results of students with high levels of mathematical ability (T)

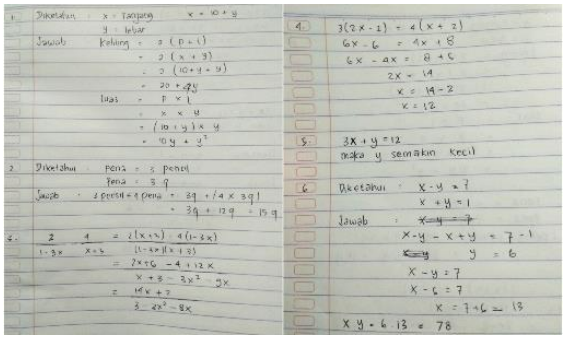

Figure 1. Student Algebra Thinking Ability Test Results

Based on the test results in Figure 1, the algebraic thinking ability of students with high mathematical ability (T) on generational ability indicators in questions number 1 and 2 shows that students understand the formula of area and circumference of rectangles and are able to model between rectangular length and square width length in the form of algebra, this shows that T students are able to determine the meaning of variables of a problem. Student T is also able to represent algebraic form in variable q as the total price to be paid well, this shows that student T is able to represent problems in the relationship between variables.

The students' algebraic thinking ability on the transformational ability indicator in questions number 3 and 4 shows that students are able to solve the algebraic fraction operation problem given by using the equalizer system. But in problem number 4 there is a mistake when determining the solution of the equation ie$2x = 14$, it should be $x = 14/2$ but student T completes with $x = 14 - 2$.

The students' algebraic thinking ability on the global meta-level indicators in questions number 5 and 6 shows that T students were able to predict changes in the y variable with the statement "then y gets smaller", although not explained directly in the written test but T students explained during the interview by analyzing the relationship between variable x and variable y. In question 6 student T is able to model the problem using algebra but has not been able to apply the elimination substitution method properly.

Description of the algebraic thinking ability test results of students with moderate mathematical ability (S)

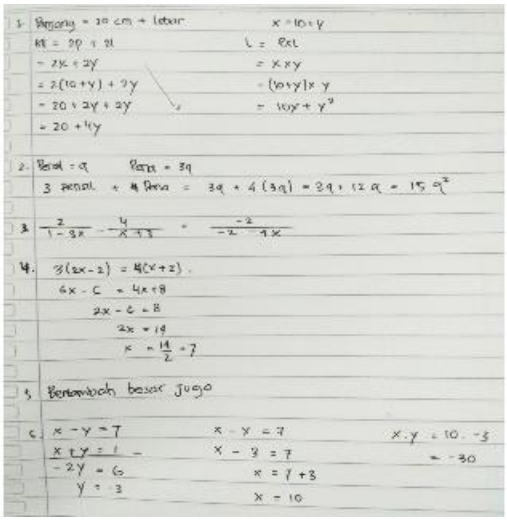

Figure 2. Student Algebra Thinking Ability Results S

Based on the test results in Figure 2, the algebraic thinking ability of students with moderate mathematical ability (S) on generational ability indicators in questions number 1 and 2 shows that S students are able to represent the perimeter problem and area of the rectangle related to the relationship between variable length and wide. S students are also able to represent algebraic form in the variable q as the total price to be paid, but at the end of the problem solving there is a mistake that is $3q + 12q = 15q^2$ should only be $15q$ without squared because it's a sum not multiplication.

S student's algebraic thinking ability on the transformational indicators in questions number 3 and 4 shows that S students have not been able to solve the algebraic fraction operation problem that was given, because S students passed the steps that should have been used to solve the problem. In problem 4 student S is able to determine the solution of the algebraic form equations that are given well and use the right steps.

S students' algebraic thinking skills on the global meta-level indicators in questions 5 and 6 show that S students have not been able to predict changes in the y variable but provide answers that almost lead to correct answers. In question number 6 S students are able to model problems using algebra, but have not been able to apply the method of substitution and elimination well so they don't find the answer.

Description of the algebraic thinking ability test results of students with low mathematical ability (R)

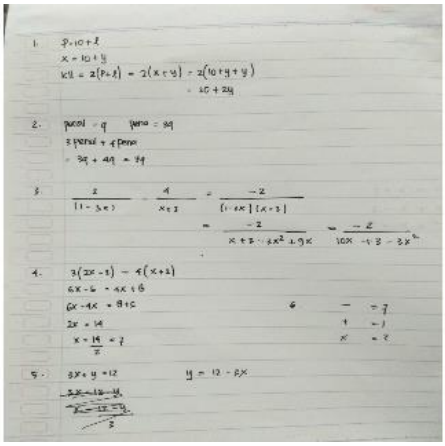

Figure 3. Student Algebra Thinking Ability Test Results R

Based on the test results in Figure 3, the algebraic thinking ability of students with low mathematical ability (R) on generational ability indicators in questions number 1 and 2 shows that student R is able to represent the relationship between the length and width of the rectangle, but is still wrong in doing operations on algebraic $2\,(10 + y + y)$ should be $20 + 4y$ not $20 + 2y$ and student R does not continue to search for the area of the rectangle. In question number 2 student R is able to represent algebraic form in the rupiah q variable as price, but it is wrong to substitute pen prices, where student S writes $3q + 4q$ should be $3q + (4x3q)$.

The students' ability to think algebraic R on the transformational indicators in questions number 3 and 4 shows that R students have almost solved the problem of the algebraic fraction form well but it is not yet correct in equating the denominator. In question number 4 student R is able to determine the completion of algebraic form equations well, using complete and appropriate steps.

The students' algebraic thinking ability on the global meta-level indicator in questions 5 and 6 shows that R students have not been able to predict changes in the y variable because they have not been able to analyze the relationship between the x and y variables in an equation $3x + y = 12$. In question 6 student R has not been able to model the problem using algebra, so he could not find the multiplication of the two numbers requested.

DISCUSSION

Based on the research objectives, this section will discuss the students' algebraic thinking abilities on algebraic material. Mathematical ability in this study is divided into three based on the level of ability, namely high, medium, and low.

Kieran (2004) argues that indicators of algebraic thinking ability in students include: (1) generational, able to determine the meaning of variables of a problem and able to represent problems in relationships between variables, (2) transformational, able to perform

algebraic form operations and able to determine the completion of an equation in algebraic form, and (3) global meta-level, able to use algebra to analyze changes, relationships and predict problems in mathematics, able to model problems and solve them. This suggests that if students are able to achieve these 3 indicators, it can be said that students have good algebraic thinking skills.

Based on the scoring criteria in table 2, T students have high generational abilities with a score of 100, this can be seen in the indicators achieved by students that are able to determine the meaning of variables of a problem and be able to represent problems in the relationships between variables.

This high level of transformational ability with a score of 80 can be seen in the indicators achieved by students which are able to solve algebraic fraction operation problems and be able to determine the completion of algebraic form equations but are not yet right in the process of operation.

The level of global meta-level capability which tends to be high with a score of 70 is seen in the indicators achieved by students that are able to predict changes in variables by analyzing the relationships between other variables and being able to represent problems in mathematical models, but is wrong in its operations.

Based on the scoring criteria in table 2, S students have high generational abilities with a score of 80 this can be seen in the indicators achieved by students that are able to determine the meaning of variables of a problem and be able to represent problems in relationships between variables.

The moderate level of transformational ability with a score of 60 can be seen in the indicators achieved by students which are able to determine the completion of the algebraic form equation but have not been able to solve the algebraic fraction operation problem.

Low to moderate global meta level capability with a score of 40 is seen in the indicators achieved by students that are able to represent problems in mathematical models, but are wrong in their operations and have not been able to predict variable changes by analyzing the relationships between other variables.

Based on the scoring criteria in table 2, R students have low to moderate generational abilities with a score of 50 this can be seen in the indicators achieved by students that are able to determine the meaning of variables of a problem and be able to represent problems in relationships between variables but are wrong in their operations.

The moderate level of transformational ability with a score of 60 can be seen in the indicators achieved by students which are able to determine the completion of the algebraic form equation but have not

been able to solve the algebraic fraction operation problem.

The level of global meta-level capability which tends to be low with a score of 30 is seen in the indicators achieved by students that are not yet able to predict changes in variables by analyzing the relationships between other variables and have not been able to represent problems in mathematical models.

In general, the cause of the lack of students' algebraic thinking skills based on the results of tests and interviews conducted is the students' lack of understanding of the properties of algebraic calculation operations so that students cannot use the rules in solving problems and the lack of question exercises conducted by students so that students' algebraic thinking abilities not well developed.

The results of this study are in line with the results of Qur'ani's research (2015) which states that there are a number of students at the junior secondary level who do not master algebra material. This research shows that there are various kinds of difficulties for students in learning algebra and in utilizing the ability to think algebra.

CONCLUSION

Based on the analysis and discussion that has been carried out from the results of research on 3 research subjects, it can be concluded that the algebraic ability of students with high levels of mathematical ability has an average value that is included in the high category, with generational abilities, transformational abilities and ability levels high global meta. Students with moderate mathematical ability have an average algebraic thinking ability that is included in the medium category, with high generational abilities, moderate transformational abilities and low to moderate global meta-level abilities. While students with low mathematical abilities have an average value of algebraic thinking skills that are included in the low category, with low to moderate generational abilities, moderate transformational abilities, and global meta-level abilities tend to be low.

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