Learning from The Didactic Transposition Process: Insights for Curriculum Improvement in Geometry Education

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ABSTRACT

This study investigated the gap between the scholarly knowledge and understanding of pre-service mathematics teachers in geometry, specifically on the topic of angle pairs formed when two lines were intersected by a transversal. This gap arose from a didactic transposition process that simplified and reorganized scholarly knowledge into knowledge to be taught (curriculum) without considering the underlying conceptual structure. The purpose of this study was to identify the form of this gap and to propose an alternative curriculum design that could strengthen students' conceptual understanding. This study used a qualitative approach with a hermeneutic phenomenological design. Data were obtained from the analysis of textbooks, teaching materials, and test results of 35 students at a state university in Aceh, Indonesia. The results showed that the existing curriculum did not emphasize the process of forming the concept of angle pairs meaningfully, but only emphasized visual recognition. As a result, students tend to understand the concept intuitively and experience difficulties when faced with representations that differ from standard illustrations as usually shown in books or lessons. This finding emphasizes the importance of designing knowledge to be taught that allows students' learned knowledge to align with scholarly knowledge. Learning from the didactic transposition process, this research provides theoretical basis and empirical example for the development of a more meaningful and coherent geometry curriculum without neglecting the axiomatic-deductive nature of geometry.

Keywords: *curriculum development, didactic transposition, geometry education.*

1. INTRODUCTION

In mathematics education, the knowledge produced within the scholarly community, such as that developed by mathematicians, does not appear in the classroom in its original form. Instead, it undergoes a transformation process that Chevallard conceptualized as didactic transposition, a sequence of changes that shifts scholarly knowledge into knowledge to be taught, which is then reshaped by teachers into taught knowledge, and eventually becomes learned knowledge in students (Chevallard & Bosch, 2020; Chevallard, 2019). Figure 1 illustrates how this transposition process operates across different institutions. This process is crucial because it shapes the structure of curriculum content, the logical flow of instructional materials, and the ways in which students come to understand mathematical concepts. A well-executed didactic transposition process ensures that the mathematical concepts delivered to students remain accurate (Jamilah et al., 2020). A poorly executed transposition process frequently leads to misconceptions, shifts in meaning, and a narrowing of students' understanding (Herizal et al., 2025b).

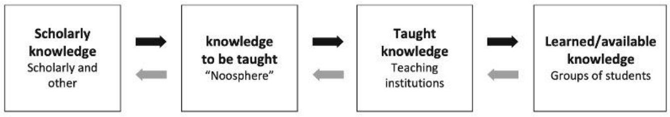


Figure 1. The Process of Didactic Transposition (Chevallard & Bosch, 2020)

Figure 1 illustrates that each stage of the didactic transposition process involves different actors. At the level of scholarly knowledge, the key contributors are mathematicians, universities, and other disciplinary institutions. The next stage involves the noosphere, a group that mediates between scientific knowledge and the educational system. This group decides how disciplinary knowledge is selected, reorganized, and adapted into the knowledge to be taught as it is officially designed by curricula. It includes scholars, educators, curriculum designers, and textbook authors. At the taught knowledge stage, teachers take on the central role, enacting the knowledge to be taught through their instructional practices in the classroom (Bosch & Gascón, 2006).

However, the process of didactic transposition does not always unfold smoothly. In practice, simplifications, omissions, or rearrangements of concepts often occur in ways that obscure the epistemological structure of the content. When such modifications are not supported by adequate pedagogical justification, their impact emerges in the form of student understanding that becomes fragmented, intuitive, and not fully grounded in the intended conceptual relationships. As an example, a research found that the way knowledge was transposed in instruction shaped how students made sense of angle definitions, and this often led them to blur the distinction between an angle as a geometric object and its measure as a numerical value (Herizal et al., 2025b). Although this issue has long been recognized within didactic studies, only a limited number of investigations have examined in detail how these transformations actually occur and what implications they hold for students' conceptual understanding.

To examine this issue more concretely, the present study uses the topic of angle pairs formed when two lines are intersected by a transversal as a case example. Although this topic is typically considered elementary in geometry, its conceptual structure is fairly intricate, involving several definitions and the relationships among geometric objects. The diversity of diagrammatic representations and the interplay among concepts make this topic an appropriate entry point for observing how changes within the didactic transposition process may shape students' understanding, particularly that of prospective mathematics teachers. In Indonesia, prospective mathematics teachers occupy a strategic position because they not only learn geometry but also learn how geometric knowledge is transformed into teachable content. When the process of didactic transposition is inaccurate or inconsistent, their understanding of a concept can become distorted, and such distortions may later influence their classroom practice. Therefore, identifying how these conceptual gaps emerge holds substantial practical value.

Moreover, research on didactic transposition remains limited within the domain of geometry, particularly for the topic examined in this study. Existing works tend to focus on isolated geometrical ideas, such as quadrilaterals (Akar & Işıksal-Bostan, 2022) or the relationship between central and inscribed angles in a circle (Winarji & Turmudi, 2020). Other studies have instead concentrated on calculus-related themes, including concave–convex functions (Strømskag & Chevallard, 2024), the fundamental theorem of calculus (Topphol, 2023), and the notions of asymptotes and asymptotic behaviour in function graphs (Katalenić et al., 2019). In this context, the present study addressed this gap by providing a detailed analysis of the didactic transposition process as it occurs in university-level mathematics instruction. Specifically, the study examined how scholarly knowledge was transformed into knowledge to be taught and knowledge actually taught, and how this transformation influenced students' understanding. By tracing these processes and identifying the resulting learning obstacles, the findings offer empirically grounded insights that may support the development of better curriculum improvements.

Building on this context, the present study aimed to analyze how scholarly knowledge on the concept of angles formed by a transversal was transformed through the process of didactic transposition, and to identify the types of gaps that emerged between scholarly knowledge and the knowledge understood by students. Using this topic as a case example, the study also sought to offer insights for improving geometry curriculum design so that the transformation of knowledge could better preserve conceptual coherence.

To achieve these aims, the study formulated three research questions: (1) how was scholarly knowledge on angle concepts transformed across the stages of didactic transposition? (2) what conceptual gaps appeared in students' learned knowledge as a result of this transformation? and (3) how the findings from this case study could contribute to the development of a more robust geometry curriculum? Through an analysis of didactic transposition, the study was expected to provide both a theoretical and practical examples for designing a geometry curriculum that not only develops procedural skills but also focuses on conceptual understanding aligned with the geometry scholarly knowledge.

2. METHODOLOGY

2.1. Research Design

This study employed a qualitative approach using a hermeneutic phenomenological design. This design was chosen to analyze how scholarly knowledge was transformed through didactic transposition into knowledge to be taught and taught knowledge and to interpret how pre-service mathematics teachers understood angle pairs formed by a transversal.

2.2. Research Context

The study was situated at a state university in Aceh, Indonesia, involving pre-service mathematics teachers and focusing on plane geometry course, particularly in the topic of angle pairs formed by a transversal, while addressing the gap between scholarly knowledge and students' understanding arising from didactic transposition in textbooks and curricula.

2.3. Participants of the Study

The participants were 35 pre-service mathematics teachers enrolled in plane geometry course at a state university in Aceh, Indonesia. A purposive sampling technique was used to select students who had studied the concept of pairs of angles formed when any two lines were cut by a transversal and its application.

2.4. Instruments

Three instruments were used in this study. First, document analysis of geometry textbook as sources of scholarly knowledge and college textbooks to examine how angle-pair concepts are presented. Second, a geometry concept test with essay items administered to 35 pre-service mathematics teachers to assess their understanding. Third, students' written work was collected to analyze their reasoning and concept construction.

2.5. Data Analysis Techniques

Data were analyzed through document analysis and thematic coding. Textbook data were examined to identify transformations of scholarly knowledge during didactic transposition. Students' test responses were coded to identify reasoning patterns and misconceptions. Findings were interpreted using hermeneutic phenomenology to connect textbook structures with students' learned knowledge.

2.6. Data Validity

Data validity was strengthened through methodological triangulation, including textbook analysis, students' test results, and Focus Group Discussions (FGD), which allowed the findings to be cross-validated across multiple data sources.

2.7. Research Ethics

Ethical considerations have been addressed by ensuring that all participating students provided informed consent, that participation was voluntary, and that anonymity and confidentiality were maintained throughout the data collection and analysis process.

3. RESULT AND DISCUSSION

3.1. The Representation of the Concept of Angles Formed by a Transversal in Scholarly Knowledge

Figure 2 illustrates how the concept of angles formed by a Transversal was introduced in scholarly knowledge. Before reaching this concept, the idea of a transversal is first discussed. A transversal is a line that intersects two or more other lines at two or more distinct points (Lewis, 1968). From these intersections, two groups of angles are formed: the angles located between the two lines, referred to as interior angles, and the angles outside the two lines, known as exterior angles (Alexander & Koeberlein, 2020; Clemens et al., 1990; Lewis, 1968). From this set of angles, several specific angle pairs are then identified, namely corresponding angles, alternate interior angles, alternate exterior angles, same-side interior angles, and same-side exterior angles (Alexander & Koeberlein, 2020; Leonard et al., 2014; Lewis, 1968; Moise, 1990).

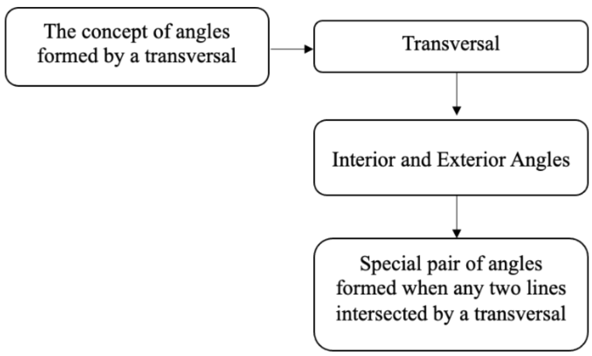


Figure 2. The Construction Process of the Concept of Angles Formed by a Transversal in Scholarly Knowledge

The introduction of special angle pairs formed when two lines are intersected by a transversal should begin with the introduction of the transversal and the interior/exterior angles (Lewis, 1968). This is because the process of identifying and defining these special angle pairs relies on these two components: the transversal and the interior/exterior angles. For example, alternate interior angles are identified by examining two non-adjacent interior angles located on opposite sides of the transversal. If students do not understand which line is the transversal or which angles are interior, it becomes difficult for them to identify these angle pairs. Even if they manage to do so, their understanding tends to be limited to memorizing specific diagrams. When the illustration changes or becomes more complex, they struggle to correctly identify the angles.

3.2. The Representation of the Concept of Angles Formed by a Transversal in Knowledge to be Taught

The data in this section were obtained from the textbooks used by the lecturers, as confirmed by the lecturers themselves. The analysis of these textbooks showed the sequence of material presentation, which is illustrated in Figure 3. The presentation began with the relationship between two lines, leading to the definition of parallel lines. Subsequently, the theorems that served as conditions for parallelism were presented, viewed in terms of the measures of special angle pairs formed when two lines were intersected by a transversal.

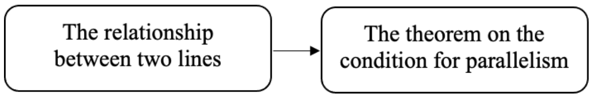


Figure 3. The Sequence of Material Presentation in The Knowledge to be Taught

A problem was identified in the structure and connection between the presented topics. The material was directly presented to the conditions for parallelism, which involved corresponding angles, alternate interior/exterior angles, and same-side interior/exterior angles. However, in the previous tasks, the definitions of these special angle pairs were not explained. This may have hindered students from properly understanding how to identify these angle pairs across different diagram representations. At least two prerequisite tasks were necessary to construct the definitions of these angle pairs: the concept of the transversal and the concept of interior/exterior angles as explained in the scholarly knowledge. These two tasks were essential in defining the special angle pairs. This finding indicated a gap in the material that disrupted the continuity of students' reasoning processes in developing a conceptual understanding of the special angle pairs formed when two lines were intersected by a transversal. This gap was caused by an obstacle called didactical obstacle, where students' difficulties arose from the way of the material was organized and introduced (Brousseau, 1997; Suryadi, 2019).

3.3. The Representation of the Concept of Angles Formed by a Transversal in Taught Knowledge

The data in this section were obtained from the analysis of teaching materials used by lecturer. Document analysis and interviews revealed that no specific teaching materials were prepared by the lecturer. In lectures, the lecturer relied on the textbooks analyzed in the previous section. Nevertheless, even when following the existing textbooks, the process of didactic transposition still occurred. Therefore, the authors examined students' notes to investigate the design of the concept of angles formed by a transversal as taught by the lecturer in class, complemented by data from lecturer interviews. Several students' notes were reviewed, revealing uniformity in terms of the sequence of topics and the characteristics of the material. Interviews with the lecturer regarding the flow of material presented in class also showed no discrepancies with the students' notes. Consequently, the authors decided to further analyze these notes as a substitute for teaching materials in order to examine the internal didactic transposition process (from knowledge to be taught to taught knowledge).

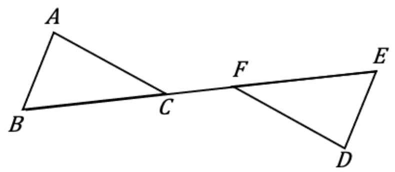
The analysis of students' notes and lecturer interviews indicated that the pattern of material presentation in class was largely similar to that found in the textbooks. The structure and development sequence followed the same order as shown in Figure 3, with no specific explanations regarding the transversal, interior/exterior angles, or the definitions of special angles formed when two lines are intersected by a transversal. This suggests a tendency for the same potential problems to arise in taught knowledge as those already present in the knowledge to be taught.

3.4. The Students' Learned Knowledge of the Concept of Angles Formed by a Transversal

To examine students' learned knowledge on the topic of the concept of angles formed by a transversal, a test was administered to them. The test results have been reported in our previous article, which showed that students' concept images differed from the formal definitions in scholarly knowledge (Herizal et al., 2025a). The study identified five main concept images: (1) pairs of alternate interior/exterior angles, corresponding angles, and same-side interior/exterior angles are formed when two parallel lines are intersected by a transversal; (2) the measures of corresponding angles must be equal, if they are not, the angles cannot be considered a pair of corresponding angles, and the same applies to alternate interior/exterior and same-side interior angles; (3) alternate exterior angles consist of a pair of angles where one angle is opposite the other and positioned outside the intersection of the lines, which can involve two transversals.

Based on the concept images obtained from the study, students' learned knowledge indicated that pairs of alternate interior/exterior angles, corresponding angles, and same-side interior/exterior angles are formed when two parallel lines are intersected by a transversal. They understood that the measures of corresponding angles, as well as alternate and same-side interior/exterior angles, must be equal to be considered proper pairs. Students also recognized that alternate exterior angles are located opposite each other outside the intersection and can involve two transversals. This form of learned knowledge indicates the existence of epistemological obstacles, specifically difficulties resulting from the initial presentation of the concept in a limited range of contexts (Brousseau, 1997).

In addition to the findings reported in the previous articles, the researchers also provided a problem to demonstrate the similarity of two triangles (see Figure 4). To solve this problem, students needed to identify congruent pairs of special angles in order to apply the Angle-Angle (A-A) theorem.



From the figure above, given that $\overline{AB} \parallel \overline{ED}$ and $\overline{AC} \parallel \overline{FD}$. Show that $\Delta ABC \sim \Delta DEF$!

Figure 4. The Problem of Angle Congruence Identification for Triangle Similarity

The test results showed that the majority of students were unable to demonstrate the similarity of the two triangles. Analysis of their responses and interviews revealed that most of their failures were not due to a lack of understanding of the similarity conditions theorem, but rather because they struggled to distinguish between parallel lines and transversals, making it difficult to identify specific congruent angle pairs. This difficulty arose because the configuration in the problem differed from the standard examples found in textbooks or presented in lectures. Furthermore, some students reported determining angle congruence solely through visual observation, without applying the properties of angles formed by parallel lines intersected by a transversal. These issues occurred because their previous coursework did not focus on the process of identifying and defining these special angle pairs. This indicates the presence of didactical learning obstacles.

3.5. Analysis of the Didactic Transposition Process

The analysis of the didactic transposition process on the concept of angles formed by a transversal revealed a noticeable gap between scholarly knowledge and students' actual understanding. Table 1 presents how the concept was introduced or understood in each institution within the didactic transposition process. In this process, the knowledge to be taught was simplified and reorganized from its original scholarly form, focusing mainly on theorems for parallel lines and measurements of special angle pairs, while leaving out explicit explanations of transversals, interior and exterior angles, and how special angle pairs are constructed. This represents an epistemological reduction, where changes in focus, sequence, and depth of content occurred as the material moved from knowledge to be taught to taught knowledge, yet the underlying conceptual weaknesses from the curriculum persisted. This phenomena was also reported in the other didactic transposition studies (Strømskag & Chevallard, 2024; Topphol, 2023).

Table 1. The Concept of Angles Formed by a Transversal in Various Institutions

The Stages of Didactic Transposition	Concept-Introduction and Understanding Approach
Scholarly Knowledge	Introduction of the concept through the transversal and the interior–exterior angles
Knowledge to be Taught	Emphasis on parallelism theorems and special-angle measurements, excluding explicit constructions of transversals, interior/exterior angles, and special angle pairs
Taught Knowledge	Approach similar to that of the knowledge to be taught
Learned Knowledge	- Identifying special angle pairs in standard cases, with difficulty connecting basic concepts to complex configurations - Special angle pairs formed by parallel lines intersected by a transversal - Requirement of equal measures for corresponding and alternate/same-side angles

As a consequence, students' learned knowledge showed conceptual fragmentation. The students could identify special angle pairs and apply definitions in standard examples, but struggled when faced with non-standard or more complex configurations. Many students had difficulty linking basic concepts, such as transversals and interior/exterior angles, to the construction of more complex angle pairs. Some also relied on visual memorization or direct observation, resulting in an understanding that was situational rather than grounded in a coherent, deep comprehension of the mathematical concepts. Although visual representations can support learning (Arcavi, 2003), depending on them too heavily can limit the development of conceptual understanding.

3.6. Insight for Curriculum Improvement

The curriculum is shaped through an agreement among various social agents, including teachers, regarding the learning activities students are expected to undertake. These activities are typically organized around disciplines, concepts, and tasks, and increasingly also encompass skills, competencies, and values (Bosch et al., 2023). All of these elements constitute the knowledge to be taught, according to the theory of didactic transposition (Chevallard & Bosch, 2020). Based on the findings from the analysis of the didactic transposition process, a revision of the geometry learning curriculum is needed, emphasizing the continuity of concepts from foundational ideas to their applications. A key suggestion is to strengthen the basic conceptual foundation, such as transversals and interior/exterior angles, before introducing students to special angle pairs. This can be achieved through gradual introduction and interactive exploration, for instance, using visual activities, manipulatives, or digital simulations, allowing students to identify angle pairs conceptually rather than merely memorizing diagrams.

Furthermore, the curriculum should explicitly emphasize the definitions and properties of special angle pairs, including corresponding angles, alternate interior/exterior angles, and same-side interior/exterior angles, across various diagrammatic representations and configurations. The goal is to enable students to apply these concepts in different contexts and develop a coherent understanding.

Overall, this proposed curriculum revision focuses on the continuity of conceptual logic, active exploration, and strengthening of foundational concepts, making the didactic transposition process more effective and helping students internalize geometric concepts in a meaningful and integrated manner.

4. CONCLUSION

This study revealed a notable gap between the formal scholarly understanding of angles formed by a transversal and the knowledge actually acquired by students. The didactic transposition process, which simplified and restructured scholarly knowledge into classroom teaching materials, often overlooked essential conceptual elements such as the definition of transversals, interior and exterior angles, and how special angle pairs are formed. Consequently, students frequently relied on memorized diagrams and visual cues, which limited their ability to apply concepts in non-standard situations.

The findings highlight the need for a geometry curriculum that prioritizes conceptual continuity, building from foundational ideas to more complex applications. By reinforcing basic concepts, explicitly introducing the properties and definitions of special angle pairs, and incorporating interactive, reflective learning experiences, students can develop a more coherent and flexible understanding. Such an approach ensures that students' learning is not only accurate in standard exercises but also transferable to diverse problem-solving contexts, bridging the gap between what is taught and what is truly understood.

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